12.5 Videos Guide

12.5a

- Equations of a line in space parallel to the vector $\mathbf{v} = \langle a, b, c \rangle$ and containing the point (x_0, y_0, z_0)
  - Vector equation: $\mathbf{r}(t) = \langle x_0, y_0, x_0 \rangle + t\langle a, b, c \rangle$
  - Parametric equations: $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$
  - Symmetric equations: $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

Exercise:

- Find a vector equation and parametric equations for the line through the point $(6, -5, 2)$ and parallel to the vector $\langle 1, 3, -\frac{2}{3} \rangle$.

12.5b

- Vector equation of a line segment from $P(x_0, y_0, z_0)$ to $Q(x_1, y_1, z_1)$:
  Let $\mathbf{r}_0(t) = \langle x_0, y_0, z_0 \rangle$ and $\mathbf{r}_1(t) = \langle x_1, y_1, z_1 \rangle$
  - $\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1, \qquad 0 \le t \le 1$

12.5c

- Equation of a plane with normal vector $\mathbf{n} = \langle a, b, c \rangle$ and containing the point (x_0, y_0, z_0)
  - $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Exercises:

- Visually represent the portion of the plane $3x + 4y + 6z = 12$ that is in the first octant.

12.5d

- Find an equation of the plane through the points $(3, 0, -1)$, $(-2, -2, 3)$, and $(7, 1, -4)$.

12.5e

- Find an equation of the plane that passes through the point $(6, -1, 3)$ and contains the line with symmetric equations $x/3 = y + 4 = z/2$.
- Where does the line through $(-3, 1, 0)$ and $(-1, 5, 6)$ intersect the plane $2x + y - z = -2$?

12.5f

- (a) Find parametric equations for the line of intersection of the planes and (b) find the angle between the planes.
  $x + y + z = 1, \qquad x + 2y + 2z = 1$

12.5g

- Distance from a point (x_1, y_1, z_1) to a plane with equation $ax + by + cz + d = 0$

- $D = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

Exercises:

- Find the distance from the point $(-6, 3, 5)$ to the plane $x - 2y - 4z = 8$.

12.5h

- Find the distance between the skew lines with the given parametric equations.

$x = 1 + t$ and $x = 1 + 2s$
$y = 1 + 6t$ $y = 5 + 15s$
$z = 2t$ $z = -2 + 6s$